 LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

 **M.Sc.** DEGREE EXAMINATION - **MATHEMATICS**

FOURTH SEMESTER – APRIL 2012

# MT 4810 - FUNCTIONAL ANALYSIS

 Date : 16-04-2012 Dept. No. Max. : 100 Marks

 Time : 1:00 - 4:00

**Answer ALL questions: (5 x 20 = 100 Marks)**

1. a) Show that every element of X/Y contains exactly one element of Z where Y and Z are

 complementary subspaces of a vector space X.

 (OR)

 If , prove that the null space has deficiency 0 or 1 in a vector space X.

 Conversely, if Z is a subspace of X of deficiency 0 or 1, show that there is an 

 such that . (5)

 b) Prove that every vector space X has a Hamel basis and all Hamel bases on X have the

 same cardinal number. (6+9)

 (OR)

 Let X be a real vector space, let Y be a subspace of X and be a real valued function

 on X such that and  for  If f

 is a linear functional on Y and prove that there is a linear

 functional F on X such that  and  (15)

1. a) Let X and Y be normed linear spaces and let T be a linear transformation of X onto

 Y. Prove that T is bounded if and only if T is continuous.

 (OR)

 If is an element of a normed linear space X, then prove that there exists an

  such that  and . (5)

b) State and prove Hahn Banach Theorem for a Complex normed linear space.

 (OR)

 State and prove the uniform Boundedness Theorem. Give an example to show that the

 uniform Boundedness Principle is not true for every normed vector space. (9+6)

1. a) Let X and Y be Banach spaces and let T be a linear transformation of X into Y. Prove that if the graph of T is closed, than T is bounded.

 (OR)

If x1 is a bounded linear functional on a Hilbert space X, prove that there is a unique

such that . (5)

b) If M is a closed subspace of a Hilbert space X, then prove that every x in X has a

 unique representation  where .

 (OR)

 State and prove Open Mapping Theorem. (15)

1. a) If T is an operator on a Hilbert space X, show that T is normal its real and imaginary parts commute.

 (OR)

If and  are normal operators on a Hilbert space X with the property that either

commute with adjoint of the other, prove that and are normal.

b) (i) If T is an operator on a Hilbert space X, prove that 

 (ii) If M and N are closed linear subspaces of a Hilbert space X and if P and Q are projections

 on M and N, then show that  (6+9)

 (OR)

State and prove Riesz-Fischer Theorem. (15)

1. a) Prove that the spectrum of  is non-empty.

 (OR)

Show that given by  is continuous, where G is the set of regular

elements in a Banach Algebra. (5)

b) State and prove the Spectral Theorem.

 (OR)

Define spectral radius and derive the formula for the same. (15)

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